# Efficient Analysis of Waveguide Filters by the Integral Equation Technique and the BI-RME Method

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*Abstract*— **This paper presents the study of rectangular waveguide filters with rounded corners in the cross–section of the waveguides. These components are suitable to low–cost mass production and can be rigorously analyzed by efficient CAD tools. The analysis approach described in this paper is based on the integral equation technique in conjunction with the Boundary Integral–Resonant Mode Expansion method. Two representative examples are also reported.**

## I. Introduction

Modern systems for mass production of waveguide components are based on low–cost fabrication techniques, such as computer–controlled milling, spark–erosion, electro–forming, or die casting. These techniques usually introduce internal rounded corners when applied to the production of passive devices in rectangular waveguide technology [1]. Taking into account these small fabrication inaccuracies in a rigorous way during the analysis stage of computer–aided design (CAD) tools would highly improved the design accuracy, reduce the development costs and time, and enhance the production yield.

The presence of rounded corners in H– and E–plane rectangular waveguide filters has been widely studied in the literature, following different techniques [2]–[4].

In this paper, we focus on the study of rectangular waveguide filters with rounded corners in the cross–section of the waveguides (see Fig. 1). Such filters represent a practical case of fabrication effects that has not been considered yet. Moreover, these structures present a two–fold advantage: on the one hand, they can be machined by low–cost techniques, and, on the other hand, they can be rigorously simulated by the efficient CAD tool described in this paper.

The analysis method described in this paper is based on the segmentation technique: the filter is segmented into elementary blocks, which are step discontinuities

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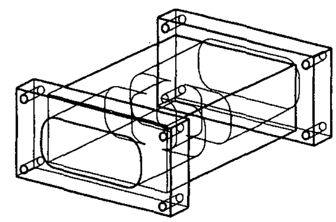

Fig. 1. Building–block of a rectangular waveguide filter with rounded corners in the cross–section of the waveguides.

between two waveguides with arbitrary cross–sections. The step discontinuities are characterized by their generalized impedance matrices, which are obtained by an integral equation technique. The Boundary Integral–Resonant Mode Expansion (BI–RME) method is used for determining the mode spectrum of the arbitrary waveguides, and novel line–integral expressions are adopted for the fast computation of modal coupling integrals involved in the calculation of the impedance matrices. Finally, the impedance matrices are properly connected: the resulting banded linear system is then solved by an efficient inversion technique. Two examples are reported, both of them showing the novelty of these structures and fully validating the analysis code.

## II. Theory

### A. Analysis of Waveguide Step Discontinuities

Many waveguide filters can be considered as the cascade of step discontinuities between two waveguides with arbitrary cross–sections (Fig. 2). The electrical behaviour of such discontinuities can be characterized through multimode equivalent circuits, such as the generalized scattering matrix, the generalized admittance matrix, or the generalized impedance matrix, which relate modal currents and voltages at the ports of the junction.

Our approach is based on the generalized impedance matrix (GIM), obtained by the integral equation



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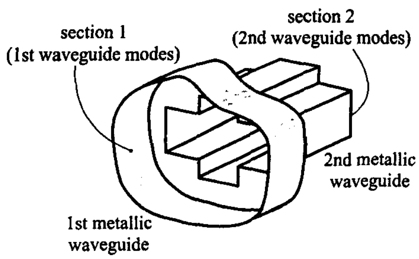


Fig. 2. Step discontinuity between two hollow waveguides with arbitrary cross–sections.

method described in [5]. The practical implementation of this method requires the knowledge of the mode spectum of the waveguides, as well as the coupling coefficients between the sets of waveguide modal vectors.

## *B. Calculation of Waveguide Modes and Coupling Coefficients*

The calculation of the mode spectrum of a waveguide with an arbitrary cross–section can be efficiently performed by the BI–RME method [3], [4], [6]. The BI–RME method provides all the waveguide modes in a prescribed frequency band by solving a single matrix eigenvalue problem involving a small number of unknowns, and was implemented in a very efficient computer code.

The calculation of the coupling integrals between the modes of two waveguides

$$\mathcal{I}_{nq} = \int_{S_2} \vec{e}_{1n} \cdot \vec{e}_{2q} \, dS \qquad (1)$$

(where $\vec{e}_{1n}$ and $\vec{e}_{2q}$ are the electric modal vectors of the larger and the smaller waveguide, respectively, and $S_2$ is the cross–section of the smaller waveguide) can be performed in two different ways.

If the larger waveguide is rectangular or circular, the coupling coefficients can be obtained as a by–product of the BI–RME calculation of the mode spectrum, as already discussed in [7].

If the larger waveguide is arbitrary, this technique can still be applied. Nevertheless, in this case, fictitious exterior rectangular waveguides must be used (see $\Omega_1$ and $\Omega_2$ in Fig. 3), and the coupling integrals between two arbitrary waveguides are converted into a double summation, which involves the coupling integrals between an arbitrary waveguide and the exterior rectangular waveguide [8]

$$\mathcal{I}_{nq} = \sum_{ij} \int_{S_1} \vec{e}_{1n} \cdot \vec{\mathcal{E}}_{1i} \, dS \int_{\Omega_2} \vec{\mathcal{E}}_{1i} \cdot \vec{\mathcal{E}}_{2j} \, dS \int_{S_2} \vec{e}_{2q} \cdot \vec{\mathcal{E}}_{2j} \, dS \qquad (2)$$

where $\vec{\mathcal{E}}_{1i}$ and $\vec{\mathcal{E}}_{2j}$ are the electric modal vectors of the rectangular waveguides embedding $S_1$ and $S_2$, respectively, and $\Omega_2$ is the cross–section of the smaller rectangular waveguide.

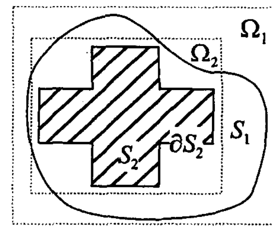


Fig. 3. Integration domain for the calculation of the coupling integrals.

In this case, it is convenient to adopt a different approach. In fact, by applying the properties of the modal vectors, the coupling coefficients can be converted from surface integrals into line integrals on the boundary $\partial S_2$ of the smaller waveguide [9]. These line integrals can be easily evaluated by a one–dimension numerical integration. Expressions reported in [9], however, are not applicable when the cutoff frequency of the two waveguide modes coincides. This case, far from being rare, prevents the use of such formulas in fully automatic computer codes. Recently, some of the authors developed new formulas which overcome this problem and are valid also when the cutoff frequency of the two modes are very close or even coincident [10]. In particular, it resulted

$$\begin{aligned}\mathcal{I}_{nq}^{TM-TM} &= \int_{S_2} \vec{e}_{1n}^{\,TM} \cdot \vec{e}_{2q}^{\,TM} \, dS \qquad (3)\\ &= \frac{\kappa_{1n}}{\kappa_{2q}(\kappa_{1n}^2 - \kappa_{2q}^2)} \int_{\partial S_2} \psi_{1n} \frac{\partial \psi_{2q}}{\partial n} \, d\ell \qquad (\kappa_{1n} \neq \kappa_{2q})\\ &= \frac{1}{\kappa_{2q}(\kappa_{1n} + \kappa_{2q})} \int_{\partial S_2} r \frac{\partial \psi_{1n}}{\partial r} \frac{\partial \psi_{2q}}{\partial n} \, d\ell \qquad (\kappa_{1n} = \kappa_{2q})\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{nq}^{TM-TE} &= \int_{S_2} \vec{e}_{1n}^{\,TM} \cdot \vec{e}_{2q}^{\,TE} \, dS \qquad (4)\\ &= -\frac{1}{\kappa_{1n}\kappa_{2q}} \int_{\partial S_2} \frac{\partial \psi_{1n}}{\partial \ell} \phi_{2q} \, d\ell\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{nq}^{TE-TE} &= \int_{S_2} \vec{e}_{1n}^{\,TE} \cdot \vec{e}_{2q}^{\,TE} \, dS \qquad (5)\\ &= \frac{\kappa_{2q}}{\kappa_{1n}(\kappa_{2q}^2 - \kappa_{1n}^2)} \int_{\partial S_2} \frac{\partial \phi_{1n}}{\partial n} \phi_{2q} \, d\ell \qquad (\kappa_{1n} \neq \kappa_{2q})\\ &= \frac{-\kappa_{2q}}{\kappa_{1n}^2(\kappa_{1n} + \kappa_{2q})} \int_{\partial S_2} \frac{\partial}{\partial n}\left(r \frac{\partial \phi_{1n}}{\partial r}\right) \phi_{2q} \, d\ell \qquad (\kappa_{1n} = \kappa_{2q})\end{aligned}$$

$$\mathcal{I}_{nq}^{TE-TM} = \int_{S_2} \vec{e}_{1n}^{\,TE} \cdot \vec{e}_{2q}^{\,TM} \, dS = 0 \qquad (6)$$

where: $\phi$ and $\psi$ are the scalar potentials of the TE and TM modes, respectively; $\kappa$ represents the cutoff

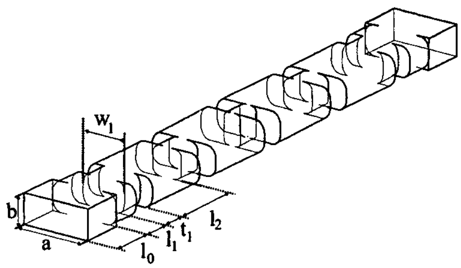


Fig. 4. Geometry of the inductively coupled rectangular waveguide filter with rounded corners. Dimensions in *mm*: $a$=22.86, $b$=10.16, $l_1$=4.00, $l_2$=14.29, $l_3$=15.844, $t_1$=1.70, $t_2$=1.77, $t_3$=1.782, $w_1$=10.50, $w_2$=6.70, $w_3$=6.15. The filter presents three simmetries and the radius of the rounded corners is $R$=2.00 *mm*.

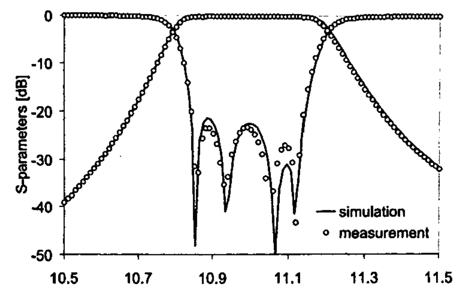


Fig. 5. Simulation and measurement of the scattering parameters of the inductively coupled rectangular waveguide filter with rounded corners, using 180 localized modes and 5 accessible modes.

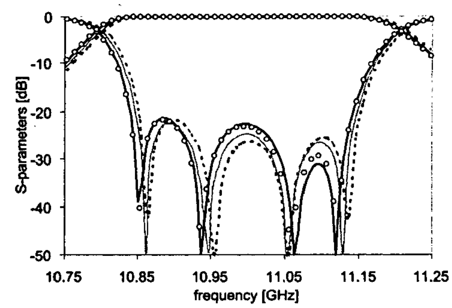


Fig. 6. Study of convergence of simulations reported in Fig. 5: analyses were performed with 40 (dashed line), 90 (solid gray line), 180 (solid black line), and 250 (markers) localized modes.

wavenumber; subscripts 1 and 2 denote the larger and the smaller waveguide, respectively; $\partial/\partial n$ and $\partial/\partial \ell$ represents the normal and the tangential derivatives on $\partial S_2$; finally, $r$ is the distance from an arbitrary point to the coordinate system origin, and $\partial/\partial r$ is the correponding radial derivative.

### C. Efficient Solution of the Banded Linear System

Once the mode spectrum and the coupling integrals have been computed, the GIM representations of all discontinuities are easily deduced. Then, in order to analyze the complete device, the GIMs must be properly connected thus giving place to a banded linear system. This system can be efficiently solved following the procedure described in [11].

## III. Examples

The first example is an inductively coupled rectangular waveguide filter with rounded corners (Fig. 4) [8]. The filter is intended for operation at 11 *GHz* with a bandwidth of 300 *MHz*. The input and output waveguides are rectangular (standard WR90), whereas all other waveguide sections present rounded corners. The structure consists of 2 transitions from rectangular to rounded waveguide, and transitions between 4 rounded cavities and their coupling inductive windows.

The analysis of this filter in 100 frequency points required 252 *sec* on a PC Pentium III @ 1 *GHz*, by exploiting the symmetries of the structure. Fig. 5 shows the comparison between simulated and measured scattering parameters. The agreement is excellent over the whole band of interest. The convergence was achieved using 180 localized modes and 5 accessible modes. Fig. 6 shows the study of the convergence by varying the number of localized modes, while keeping 5 accessible modes.

It is worth observing that this filter presents a very low insertion loss in the pass band (0.15 *dB*), which can be attributed to its manufacturing process. The filter has been built in 5 building blocks (see Fig. 1), whose input and output ports have been deliberately defined at the middle plane of each resonant cavity. Due to the narrow bandwidth of the filter, the field pattern inside each resonator is almost coincident with the one of the fundamental resonant mode of a trirectangular cavity. Thus, on the cut plane, the magnetic field is longitudinal and, therefore, the surface current density is transverse. Hence, the effect of the cutting on the current is negligible, thus avoiding losses due to not perfect contact between the building blocks of the filter.

The second example is a rectangular waveguide filter with tuning elements (Fig. 7) [12]. The filter operates at 13 *GHz* with a bandwidth of 300 *MHz*. The structure consists of a rectangular waveguide (standard WR75) with five inductive irises, which create four cavities. Tuning elements are used for adjusting the resonance frequency of the cavities (square screws, 4 by 4 $mm^2$) as well as for tuning the coupling (square screws, 2 by 2 $mm^2$). The filter analysis performed by our code required 115 *sec* on a PC Pentium III @ 1 *GHz*, by exploiting the symmetries of the structure (Fig. 8). The

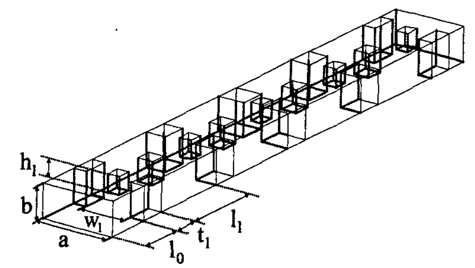


Fig. 7. Geometry of the rectangular waveguide filter with square tuning elements. Dimensions in *mm*: $a$=19.05, $b$=9.525, $l_1$=10.5, $l_2$=13.3, $t_1$=2.0, $t_2$=2.0, $t_3$=2.0, $w_1$=8.7, $w_2$=5.1, $w_3$=5.1, $h_1$=0.792, $h_2$=1.816, $h_3$=2.141, $h_4$=0.456, $h_5$=0.986. The filter presents two simmetries.

convergence was achieved using 120 localized modes and 20 accessible modes.

Fig. 9 shows the effects due to rounded corners (with different radius) on the performance of this filter. Due to the rounded corners, the pass band of the filter is narrowed and moved up in frequency. Of course, it is possible to reoptimize the filter, by properly adjusting the depth of the screws, in order to recover the original performance.

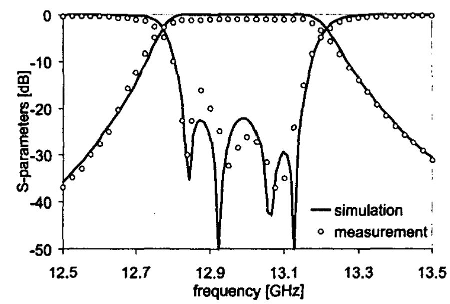


Fig. 8. Simulation and measurement of the scattering parameters of the rectangular waveguide filter with square tuning elements.

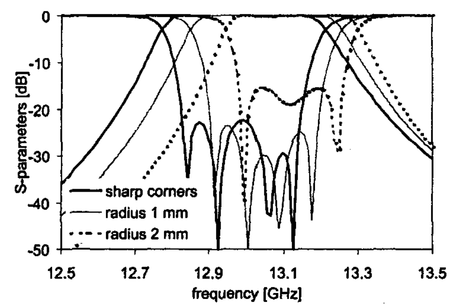


Fig. 9. Effect of rounded corners on the performance of filter in Fig. 7. Radii of 1 *mm* and 2 *mm* have been considered.

## IV. Conclusion

This paper has presented the analysis of waveguide filters with rounded corners in the cross–section of the waveguide. A novel CAD tool, based on the integral equation technique and the Boundary Integral–Resonant Mode Expansion method, has been devoloped for the rigorous analysis of these structures. A prototype operating in the X–band has demonstrated the feasibility of these filters by standard fabrication techniques. Finally, an inductive filter with tuning elements has been analyzed and compared with experimental results. Moreover, a parametric study has shown the effect of rounded corners on the performance of this filter.

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